Name: ______________________

Pid: ______________________

Show all of your work. Full credit will be given only for answers with explanations.

1. (100 points) Check all the correct statements.
   - ◯ $u \cdot v = -7$, where $u = \langle 1, 2, 7\rangle$ and $v = \langle 4, -2, -1\rangle$.
   - ◯ Length of the projection of the vector $\langle 2, 2, 7\rangle$ on the line going throw the vector $\langle 3, 6, 2\rangle$ is equal to $\frac{32}{49}$
   - ◯ The angle between the vector $\langle 1, 1, 1\rangle$ and $\langle 1, 1, 0\rangle$ is equal to $\arccos \frac{2}{\sqrt{6}}$.
   - ◯ $u \times v = w$, where $u = \langle 1, 1, 0\rangle$, $v = \langle 1, 2, 0\rangle$ and $w = \langle 1, -1, 0\rangle$.
   - ◯ The vector $\langle 1, 3, 5\rangle$ is the direction of the line defined by the equation

$$\frac{x-1}{2} = \frac{y-3}{3} = \frac{z-5}{4}.$$

Solution:

1. $u \cdot v = 1 \cdot 4 + 2 \cdot (-2) + 7 \cdot (-1) = 4 - 4 - 7 = 7$. Hence, the statement is true.
2. Length of the projection of the vector $u = \langle 2, 2, 7\rangle$ on the line going throw the vector $v = \langle 3, 6, 2\rangle$ is eual to $\frac{u \cdot v}{|v|} = \frac{2\cdot 3+2\cdot 6+7\cdot 2}{\sqrt{3^2+6^2+2^2}} = \frac{32}{7}$. Hence, the statement is not true.
3. Let $u = \langle 1, 1, 1\rangle$ and $v = \langle 1, 1, 0\rangle$. Note that the angle between these two vectors is equal to $\arccos \frac{u \cdot v}{|u| \cdot |v|} = \frac{2}{\sqrt{3}\sqrt{2}}$. Hence, the statement is true.
4. $u \times v = (1 \cdot 0 - 2 \cdot 0)i - (1 \cdot 0 - 0 \cdot 0)j + (1 \cdot 2 - 1 \cdot 1)k = k$.
5. The statement is not true, since the denominators should be equal to the components of the direction of the line.

2. Let $A = \langle 2, 0, 0\rangle$, $B = \langle 0, 4, 0\rangle$.

(a) (10 points) Find a direction vector of the line that goes through the points A and B.

Solution: Note that the line goes in the direction $\vec{AB} = \langle -2, 4, 0\rangle$.

(b) (10 points) Find a parametric form of the line that goes through the points A and B.

Solution: The parametric form of a line is $r = r_0 + tv$ where v is the dirrection and r_0 is some point from the line. Hence, the parametric form of the line that goes throw the points A and B is $r = \langle -2t, 4 + 4t, 0\rangle$.

(c) (10 points) Find an equation of the line that goes through the points A and B.

Solution: The euqation of the line is $\begin{cases} -\frac{x}{2} = \frac{y-4}{4} \\ z = 0 \end{cases}$ since the parametric form of the line is $\langle x, y, z\rangle = \langle -2t, 4 + 4t, 0\rangle$.

3. (10 points) Find $u \times v$, where $u = \langle 1, 1, 0\rangle$, $v = \langle 1, 0, 1\rangle$

Solution: Note that $u = i + j$ and $v = i + k$. Hence, $u \times v = i \times k + j \times i + j \times k = -j - k + i = \langle 1, -1, -1\rangle$.

4. Let $A = \langle 1, -1, 2\rangle$, $B = \langle -1, 0, 1\rangle$, and $C = \langle 0, 2, 1\rangle$.

(a) (10 points) Find a vector n which is perpendicular to the plane that goes through the points A, B, and C.

Solution: Let $u = \vec{AB} = \langle -1-1, 0+1, 1-2\rangle = \langle -2, 1, -1\rangle$ and $v = \vec{AC} = \langle 0-1, 2+1, 1-2\rangle = \langle -1, 3, -1\rangle$. Note that we just need to find a vector n that is perpendicular to both u and v. Recall that $u \times v$ is perpendicular to both u and v. Hence, may just choose $n = u \times v$. Hence, the result is $\langle -2, 1, 1\rangle \times \langle -1, 3, -1\rangle = (1 \cdot (-1) - 3 \cdot (-1))i - ((-2) \cdot (-1) - (-1) \cdot (-1))j + ((-2) \cdot 3 - 1 \cdot (-1))k = 2i - j - 5k = \langle 2, -1, -5\rangle$.

(b) (10 points) Find the equation of the plane passing through the points A, B, and C.

Solution: Note that a vector $v = \langle x, y, z\rangle$ is perpendicular to n iff $v \cdot n = 0$. In other words a point P belongs to the plane iff $\vec{AP} \cdot n = 0$. As a result, the equation of the plane is $2(x-1) - (y+1) - 5 \cdot (z-2) = 0$.