Name: ____________________

Pid: ____________________

1. Find P- and N-positions in the misére subtraction game with the subtrucation set $\{1, 3, 5\}$.

Solution:

2. Two players are playing the following combinatorial game.

   - On each turn they put a chess knight on a board 9×9 so that it is not attacked by previously placed knights.
   - The take turns and the player that cannot make a move loses.

   Determine who has a winning strategy.

   Solution:

3. Two players are playing the following combinatorial game with several piles of chips.

   - They take turns;
   - on each turn the current player splits one pile into two non-empty piles of chips;
   - the player that cannot make a move loses.

   Find the value of the Sprague–Grundy function for positions with one pile consisting of n chips.

   Solution:

4. We say that $\bar{L} = (\Omega, \Pr)$ is a ranomized B-decision list iff Ω is a finite set of B-decision lists and Pr is a probability distribution on Ω.

   We define $\ell(L, x)$ recursively:

   - If L is an integer, then $\ell(L, x) = 0$.
   - If $L = (f, y, L')$, then
   $$\ell(L, x) = \begin{cases} 1 & \text{if } f(x) = 1 \\ 1 + \ell(L', x) & \text{if } f(x) = 0 \end{cases}.$$

   Let $f : [1000] \to \mathbb{Z}$; we denote by $\mathrm{RL}(f) = \min_{\bar{L}} \max_{x \in [1000]} \mathbb{E}_{L \sim \bar{L}} \ell(L, x)$.

   Show that $\mathrm{RL}(\mathrm{id}) \geq 500$, where $\mathrm{id} : [1000] \to \mathbb{Z}$ and $\mathrm{id}(x) = x$.

Solution: