

Name: \_\_\_\_\_

Pid: \_\_\_\_\_

1. (10 points) Let us consider Fano's geometry, it is a theory with undefined terms: point, line, is on, and axioms:

1. There exists at least one line.
2. Every line has exactly three points on it.
3. Not all the points are on the same line.
4. For two distinct points, there exists exactly one line on both of them.
5. Each two lines have at least one point on both of them.

Show that there are exactly seven points.

2. (10 points) Prove the following recurrent relation:

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

3. (10 points) How many pairs of subsets  $A, B \subseteq [n]$  are there such that  $A \cap B \neq \emptyset$ .

4. (10 points) Find a closed formula (no summation signs) for the expression  $\sum_{i=1}^n i^2 \binom{n}{i}$ .

5. (10 points) Let us assume that we are given  $\ell$  lines that are not parallel to each other. Prove that there are at least two of them such that angle between them is at most  $\pi/\ell$ .