Name: ______________________

Pid: ______________________

1. (10 points) Let a_n be a sequence such that $a_1 = 9$, $a_2 = 41$, and $a_{n+2} = 9a_{n+1} - 20a_n$. Show that $a_n = 4^n + 5^n$.

2. We say that L is a list of powers of x iff

   - either $L = x^k$ for some positive integer k or
   - $L = (x^k, L')$ where L' is a list of powers of x and k is a positive integer.

   Let L be a list of powers of x. We say that the sum of L with $x = v$ denoted by $\sum L\big|_{x=v}$

   - is equal to v^k whether $L = x^k$ and
   - is equal to $v^k + \sum L'\big|_{x=v}$ whether $L = (x^k, L')$.

   Prove that for any list L of powers of x there is a polynomial such that $\sum L\big|_{x=v} = p(v)$ for all real numbers v.

3. (10 points) Prove that $\sum_{i=1}^{n} (i+1)2^i = n2^{n+1}$ for all integers $n \geq 1$.