

Name: _____

Pid: _____

1. (40 points) Check all the correct statements.

- The statement $x \leq -1$ or $x \geq 1$ implies the statement $x^2 \geq 1$ for all real numbers x .
- If you can prove a statement directly, you can prove it by contradiction.
- $\sum_{i=0}^{10} i = \frac{10 \cdot 11}{2}$.
- If you can prove a statement by contraposition, you can prove it by contradiction.

2. (10 points) Let us consider three-points geometry, it is a theory with undefined terms: point, line, is on, and axioms:
1. There exist exactly three points.
 2. Two distinct points are on exactly one line.
 3. Not all the three points are collinear i.e. they do not lay on the same line.
 4. Two distinct lines are on at least one point i.e. there is at least one point such that it is on both lines.

Show that there are exactly three lines.

3. (10 points) Show that $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$ for all integers $n \geq 1$.

4. (10 points) Show that $\sum_{i=1}^n \frac{1}{i^2} \leq 2 - \frac{1}{n}$ for all integers $n \geq 1$.

5. Show that 3 divides $n^3 + 2n$.