Name: _____

Pid: _____

1. (10 points) Let us consider a signature $(I, <; 0)$, where I is a unary relation intended to mean "is interesting", $<$ is a binary relation intended to mean "is less than", and 0 is a constant (a function with zero arguments).

   Translate into this language the English sentences listed below. If the English sentence is ambiguous, you will need more than one translation.

   - Zero is less than any number.
   - If any number is interesting, then zero is interesting.
   - No number is less than zero.
   - Any uninteresting number with the property that all smaller numbers are interesting certainly is interesting.
   - There is no number such that all numbers are less than it.
   - There is no number such that no number is less than it.

2. (10 points) Let us consider a signature $\mathcal{S} = (=; +, \cdot)$, where predicates and functions are binary. Let $\mathfrak{M} = (\mathbb{N}; =; +, \cdot)$ be a structure.

   - Write a formula ϕ depending on x such that for any assignment s, $\mathfrak{M} \models \phi[s]$ iff $s(x) = 1$.
   - Write a formula ϕ depending on x and y such that for any assignment s, $\mathfrak{M} \models \phi[s]$ iff $s(x) \leq s(y)$.