

Name: _____

Pid: _____

1. (10 points) Derive $(A \wedge B) \Rightarrow C$ from $A \Rightarrow (B \Rightarrow C)$ using natural derivations.

Solution:

1		$A \Rightarrow (B \Rightarrow C)$	
2			
3			
4			
5			
6			
7			

2		$A \wedge B$	
3			
3			A $\wedge E, 2$
4			$B \Rightarrow C$ $\Rightarrow E, 1, 3$
5			B $\wedge E, 2$
6			C $\Rightarrow E, 4, 5$
7		$(A \wedge B) \Rightarrow C$	$\Rightarrow I, 1-6$

2. (10 points) Derive $A \vee B$ from $X \vee A$ and $\neg X \vee B$ using natural derivations.

Solution:

1	$x \vee A$	
2	$\neg x \vee B$	
3	A	
	\overline{A}	
4	$A \vee B$	$\vee I, 3$
5	x	
	\overline{x}	
6	$\neg x$	
	\perp	$\neg E, 6$
7	\perp	
8	$A \vee B$	$\perp E, 7$
9	B	
	\overline{B}	
10	$A \vee B$	$\vee I, 9$
11	$A \vee B$	$\vee E, 2, 6-10, 10-9$
12	$A \vee B$	$\vee E, 1, 3-4, 5-11$