Lecture 12

Definition

A prop. formula φ on Ω is a tautology iff $\varphi|_\rho = T$ for any assignement ρ to Ω.

Consider $\varphi = (\neg x \vee y) \wedge (\neg y \vee x) \wedge (\neg x \vee \neg y) \wedge$
$\wedge (x \vee y)$

x	y	φ
T	T	F
T	F	F
F	T	F
F	F	F

We need Examples!

Assumptions

- If Joe is a good Boy, ↙ $p \Rightarrow q$
  then Joe has a present for Christmas.
- Joe is a good boy ← p

Conclusion

Joe has a present. ← q

Let p denote „Joe is a good Boy"
q denote „J. has a present"

$((p \Rightarrow q) \wedge p) \Rightarrow q$ is a tautology.

Assumption

- If $x \neq 0$, then $x^2 \neq 0$ ← $\neg p \Rightarrow \neg q$

Conclusion:

if $x^2 = 0$, then $x = 0$ ← $q \Rightarrow p$

Let p denote "$x=0$"

q denote "$x^2=0$"

$(\neg p \Rightarrow \neg q) \Rightarrow (q \Rightarrow p)$ is a tautology.

$(p \rightarrow \neg q) \Rightarrow (\neg q \Rightarrow \neg p)$ is a taut.

$x \Rightarrow \neg y$

$\neg x \vee y$

Assumptions:

- if $\underbrace{x=0}_{p}$, then $\underbrace{x^2=0}_{q}$
- $\underbrace{x^2 \text{ is not equal to } 0}_{\neg q}$

Conclusion:

$x \neq 0$

if $x^2 \neq 0$, then $x \neq 0$ ← $\neg q \Rightarrow \neg p$

$x^2 \neq 0$

$x \neq 0$ ← $\neg p$

1 | $p \Rightarrow q$

2 | $\neg q$

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3 | $\neg q \Rightarrow \neg p$ by contr. and 1.

4 | $\neg p$ by modus ponens and 2, 3

Rules.

m. | $A \wedge B$

| A by E-$\wedge$ m

m | A

n | B

| $A \wedge B$ by I-$\wedge$ m, n

m $\mid$ A

$\mid$ $A \lor B$ I-$\lor$ m

g $\mid$ $A \lor B$

k $\mid$ $\mid$ A

l $\mid$ $\mid$ C

m $\mid$ $\mid$ B

n $\mid$ $\mid$ C

$\mid$ C E-$\lor$ g,

k - l

m - n

m $\mid$ $\mid$ A

n $\mid$ $\mid$ B

$\mid$ $A \Rightarrow B$ I-$\Rightarrow$, m - n

m $\mid$ A

n $\mid$ $A \Rightarrow B$

$\mid$ B E-$\Rightarrow$ m, n

$\bot \leftarrow$ contr.

$$\begin{array}{r|l} m & \bot \\ & A \end{array}$$

$$\begin{array}{r|l|l} m & & A \\ \cline{3-3} n & & \bot \\ & \multicolumn{2}{l}{\neg A \quad \text{by } I\text{-}\neg,\ m\text{-}n} \end{array}$$

$$\begin{array}{r|l|l} m & & \neg A \\ \cline{3-3} n & & \bot \\ & \multicolumn{2}{l}{A} \end{array}$$

$$\begin{array}{r|l} m & A \\ n & \neg A \\ & \bot \quad \text{by } I\text{-}\bot \quad m, n \end{array}$$