Lecture 14

$$
\begin{array}{r|l}
1 & A \wedge B \wedge C \\ \hline
2 & A \wedge B \\
3 & A
\end{array}
\qquad\qquad
\begin{array}{|l}
\hline
\vdots \\
A \vee \neg A
\end{array}
\qquad\qquad
\models
$$

Definition

Let $\varphi_1 \ldots \varphi_n, \psi$ be prop. formulas on Ω.

We say that $\varphi_1 \ldots \varphi_n$ semantically imply ψ ($\varphi_1 \ldots \varphi_n \models \psi$) iff

$$\varphi_1|_\rho \wedge \varphi_2|_\rho \ldots \wedge \varphi_n|_\rho \Rightarrow \psi|_\rho \text{ is}$$

true for any asignement ρ to Ω.

Theorem (Completeness)

Let $\varphi_1 \dots \varphi_n, \psi$ be some formulas on Ω.

Then if $\varphi_1 \dots \varphi_n \models \psi$, then we can derive ψ from $\varphi_1 \dots \varphi_n$

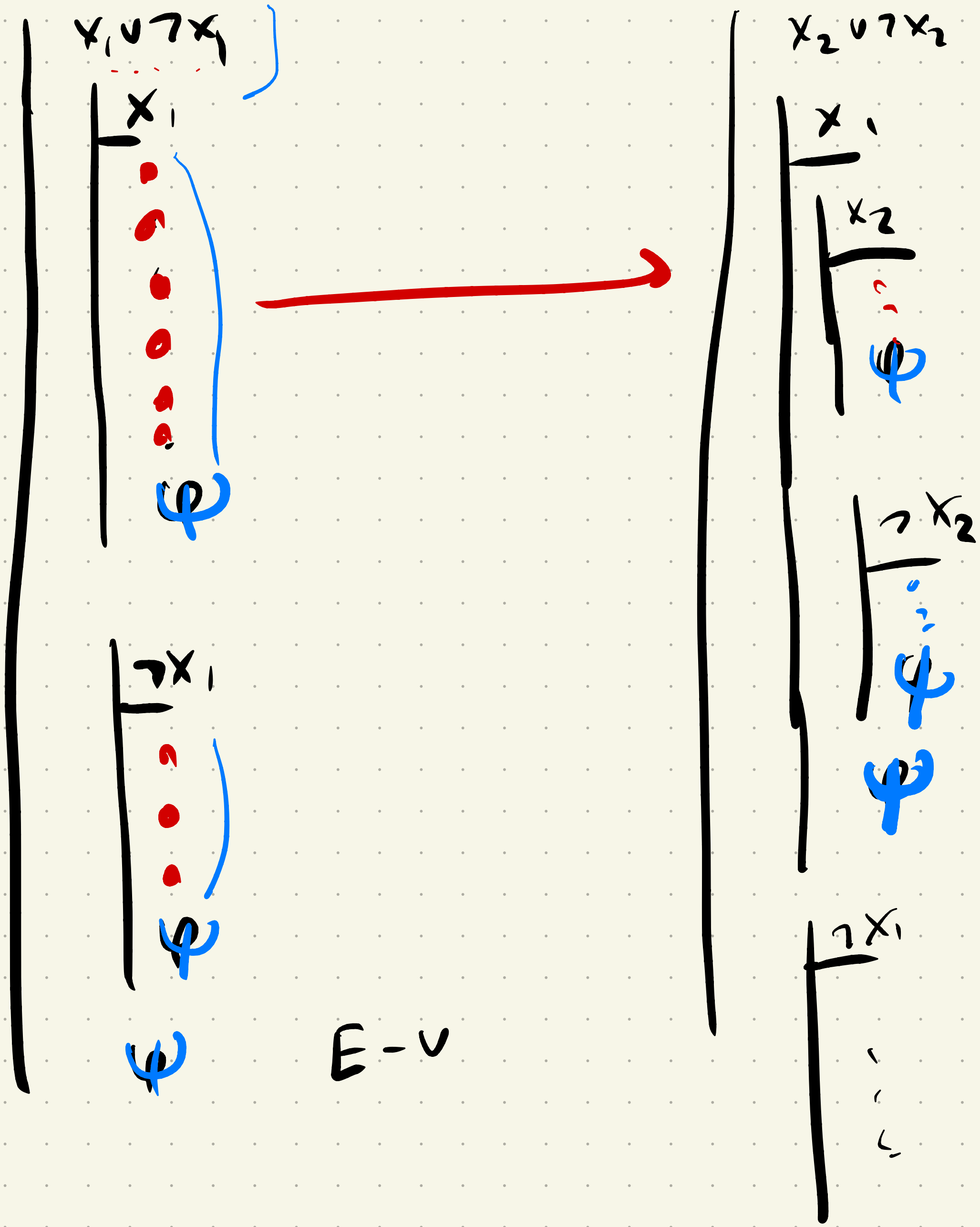
$$
\begin{array}{l|l}
1 & \varphi_1 \\
2 & \varphi_2 \\
\vdots & \vdots \\
n & \varphi_n \\
\hline
 & \vdots \\
 & \psi
\end{array}
$$

Proof WLOG Ω is finite

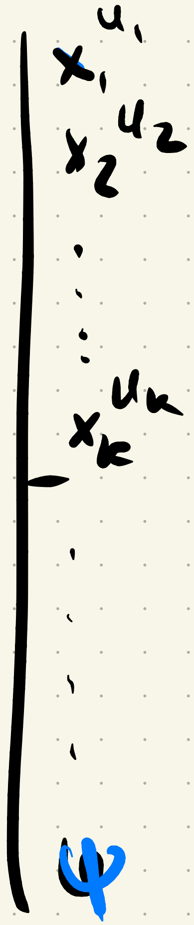
Assume that $\Omega = \{x_1 \dots x_k\}$

1. Consider $n = 0$ i.e. we need to construct

$$
\begin{array}{l|l}
\hline
1 & x_1 \vee \neg x_1 \\
2 & x_2 \vee \neg x_2 \\
\vdots & \vdots \\
k & x_k \vee \neg x_k
\end{array}
\qquad\qquad
\begin{array}{|l}
\hline
\vdots \\
\psi
\end{array}
$$



x₁ ∨ ¬x₁
x₁
φ
¬x₁
φ
φ
E-∨
x₂ ∨ ¬x₂
x₁
x₂
φ
¬x₂
φ
φ
¬x₁



Lemma

Let ψ be a prop formula on $x_1 \ldots x_n$, let $u_1 \ldots u_n \in \{T, F\}$.
s.t. $\psi|_{x_1 = u_1 \ldots x_n = u_n} = T$.
Then we can derive ψ from $x_1^{u_1}, \ldots, x_n^{u_n}$.