Lecture

Definition

Let S be a signature. Let Σ be a set of pred. formulas in S and φ be a pred. formula in S'

We say that $\Sigma \models \varphi$ iff

whenever Σ is true, φ is true as well.

For any structure in $\mathcal{M}$ and an assignement s, if $\mathcal{M} \models \Sigma\, [s]$ then $\mathcal{M} \models \varphi[s]$.

this means that $\forall \psi \in \Sigma$ $\mathcal{M} \models \psi[s]$

Let $S = (P)$

$\varphi = \forall x\ P(x)$

to answer the question you need a structure

$\varphi = P(y) \wedge \exists x\ P(x)$

$\mathcal{M} = (\mathbb{N};$ is even$)$

Consider $S = (\leq)$. Let Σ be the set of following formulas:

1) $\neg(a<a)$
2) $(a<b \wedge b<c) \Rightarrow a<c$
3) $a<b \Rightarrow \neg(b<a)$

1) $a \leq a$
2) $(a \leq b \wedge b \leq c) \Rightarrow a \leq c$

Let φ be $\exists x \forall y \; x \leq y$

Question: Is it true that $\Sigma \models \varphi$?

Note that $\mathcal{M} = (\mathbb{N}, <)$ makes Σ to be true but φ is false since $x<x$ is false

$\mathcal{M} = (\mathbb{Z}; \leq)$

notice that $\mathcal{M} \not\models \varphi$ but $\mathcal{M} \models \Sigma[s]$ for any s.

How to formulate that there are only 3 elements.

$\psi = \exists x\, y\, z\, \forall w \; \underline{w=x \vee w=y \vee w=z}$

$(w \leq x \wedge x \leq w) \vee \ldots$

Show that $\Sigma \cup \{\psi\} \models \varphi$