Lecture 4: Structural Induction

$1 + 2 \cdot 3$

$1 -$

$((1 + 2 + (3 + 4))$

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1 ✓

$(1\ 2)$ $(2\ 1)$ ✓

$(1\ (2\ 3))$ ✓ $((1\ 2)\ (2\ 3))$ ✓

$(1\ 2\ 3)$ ✗

$(1\ (2\ 3)$ ✗

Definition A binary tree is a sequence of integers and parentheses such that

(the base case) an integer is a binary tree

(recursion step) if T_1, T_2 are b.t., then $(T_1\ T_2)$ is a binary tree.

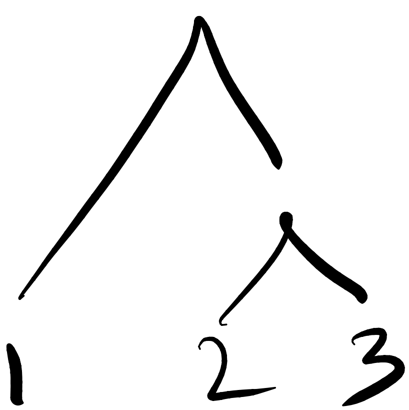
Exercise

- Is $(1\ (2\ 3))$ a binary tree?
- Is $(1\ 2\ 3)$ a binary tree?

Exercise

- Is ((1 (2 3)) a binary tree?
- Is (1 2 3) a binary tree?

123 (23) ((1 (2 3))



General case

Let U be some set

Let $B \subseteq U$

Let $\mathcal{F} = \{ f_1 : U^{l_1} \to U \ldots f_n : U^{l_n} \to U \}$

S is the set generated by $\mathcal{F}$ from B iff

$u \in S$ iff $\exists\, u_1 \ldots u_m \in U$, $u_m = u$

s.t. $\forall i \in [m]$

- $u_i \in B$
- $u_i = f_j(u_{i_1} \ldots u_{i_{l_j}})$ where $i_1 \ldots i_{l_j} < i$

Binary trees

U is a set of seq. of integers and par.

a set of seq. cons. of one integer

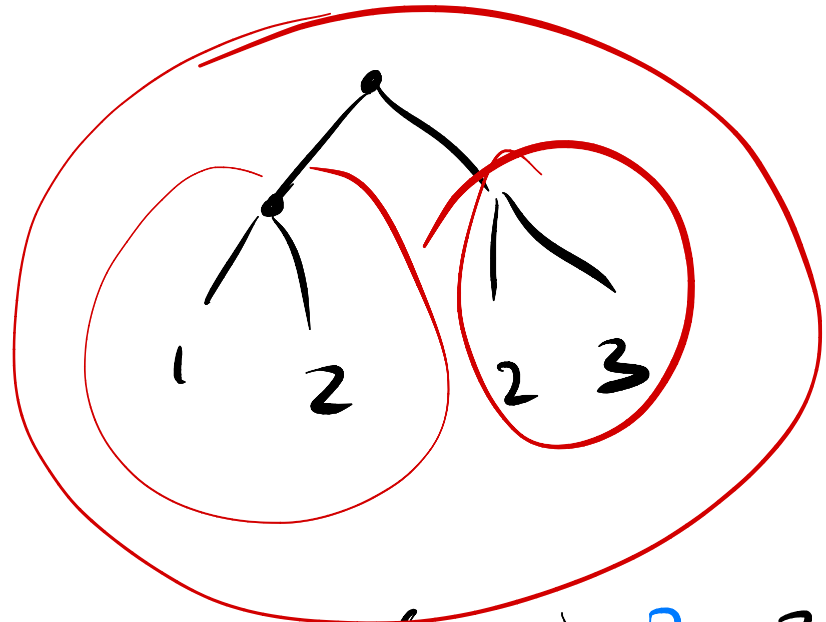
$\mathcal{F} = \{ f : U^2 \to U \}$

$f(T_1\ T_2) = (T_1\ T_2)$

Exercise

Write seq. $u_1 \ldots u_m$ for

- ((1 2) (2 3))
- ((1 2) 4)



1, 2, (1 2), 2, 3, (2 3), ((1 2) (2 3))

1, 2, 4, (1 2), ((1 2) 4)