Lecture 9: Orderings and Connectives

Let $x, y \in \mathbb{Z}$. We say that $x \mid y$ iff $xk = y$ for some $k \in \mathbb{Z}$.

Exercise.

$\mid$ defines a partial ordering on $\mathbb{Z}$.

Reflexivity: We need to check that $x \mid x$ for any $x \in \mathbb{Z}$. Clearly this is true since $x \cdot 1 = x$.

Antisymetry We need to check that if $x \mid y$ and $y \mid x$ for some $x, y \in \mathbb{Z}$, then $x = y$. We know that $xk = y$ $yl = x$

Antisymetry We need to check that if $x|y$ and $y|x$ for some $x, y \in \mathbb{Z}$, then $x=y$. We know that $xk=y$ $yl=x$; hence $xkl=x$. So either $x=0$ and $y=0$, or $kl=1$ i.e. $k=l=1$.

Transitivity We need to show that if $x|y$ $y|z$, then $x|z$

$xk=y$ $yl=z$; hence $xkl=z$

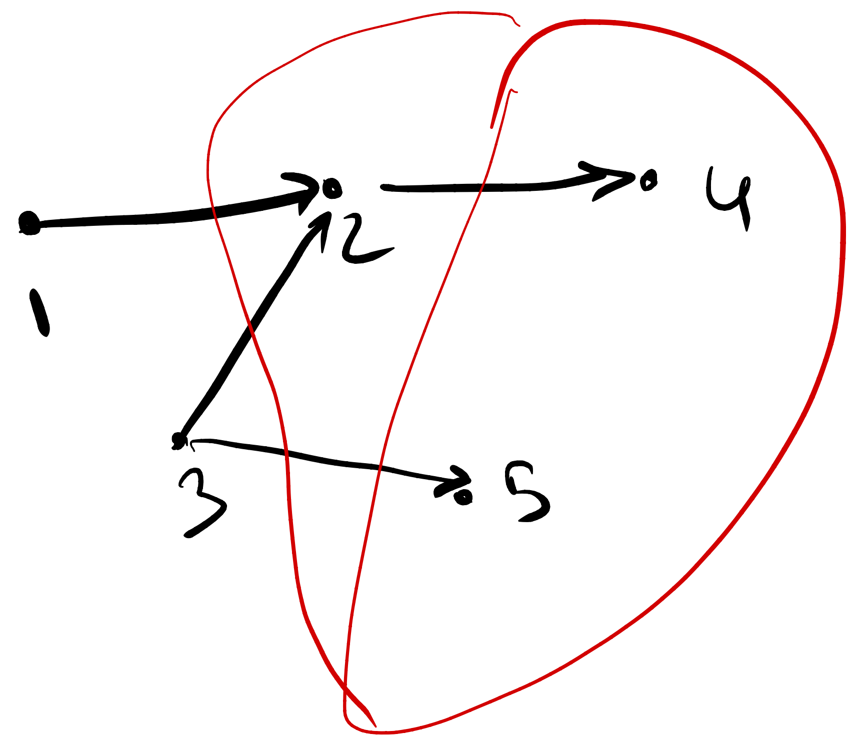
We have a list of steps in a recipe

1 Get Tomatos
2. Get mushrooms
3. Get eggs
4. Chop tomatos
5 Chop mushrooms
6. heat the pan
7. break the eggs
8 put tomatos into pan
9 put mushrooms

Theorem

Let S be some finite set and let $\preccurlyeq$ be a partial ordering of S.
Then there is a total ordering $\preccurlyeq_t$ of S s.t.

for any $x, y \in S$, if $x \preccurlyeq y$, then $x \preccurlyeq_t y$.



4 5 2 1 3

Topological sort

Let us consider the following binary operations over $\{T, F\}$

x	y	$\wedge$	$\vee$	$\Rightarrow$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

x	$\neg x$
T	F
F	T

AND - Conjunction - $\wedge$

OR - Disjunction - $\vee$

Implication - $\Rightarrow$

$\rightarrow$

$\supset$

Negation - $\neg$

$\neg$

Definition Let Ω be set of symbols

U = the set of sequences of symb. „$\wedge$", „$\vee$", „$\Rightarrow$", „$\neg$", elements of Ω.

$$\mathcal{F} = \{ f_\wedge : U^2 \to U,\ f_\vee : U^2 \to U,\ f_\Rightarrow : U^2 \to U,\ f_\neg : U \to U \}$$

$$B = \Omega$$

$$f_\wedge(\varphi_1, \varphi_2) = (\varphi_1 \wedge \varphi_2)$$
$$f_\vee(\varphi_1, \varphi_2) = (\varphi_1 \vee \varphi_2)$$

$\vdots$

The set generated by $\mathcal{F}$ from $\mathcal{B}$ is called the set of propositional formulas or Boolean formulas.