Math 160B Practice Midterm 2 February 15, 2020

Name: _____

Pid: _____

1. Let U be a Gödel Universal function, and let $S = \{n \ : \ U_n \text{ is a surjective function}\}$. Is S enumerable? Is $\mathbb{N} \setminus S$ enumerable?

2. Let U be a Universal Gödel function. Let $H = \{(n, x) \in \mathbb{N}^2 \ : \ U(n, x) \text{ terminates}\}$. Show that $H \in \Sigma_1$ and that $A \leq_m H$ for any $A \in \Sigma_1$.

3. Show that there is a number $r \in \mathbb{R}$ such that $\{q \in \mathbb{Q} \ : \ q < r\}$ is not enumerable.

4. Let A be an algorithm computing Gödel universal function. We say that the running time of A at n is polynomial iff there is an integer k such that $A(n, x)$ terminates after $k + \log^k(x)$ steps.

   Show that the set

   $$\{n \ : \text{ the running time of } A \text{ at } n \text{ is polynomial}\}$$

   is undedicable.