Name: _____

Pid: _____

1. (20 points) Prove the following equalities.

(a) $1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1/2)(n+1)}{3}$;

(b) $1^3 + 2^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$;
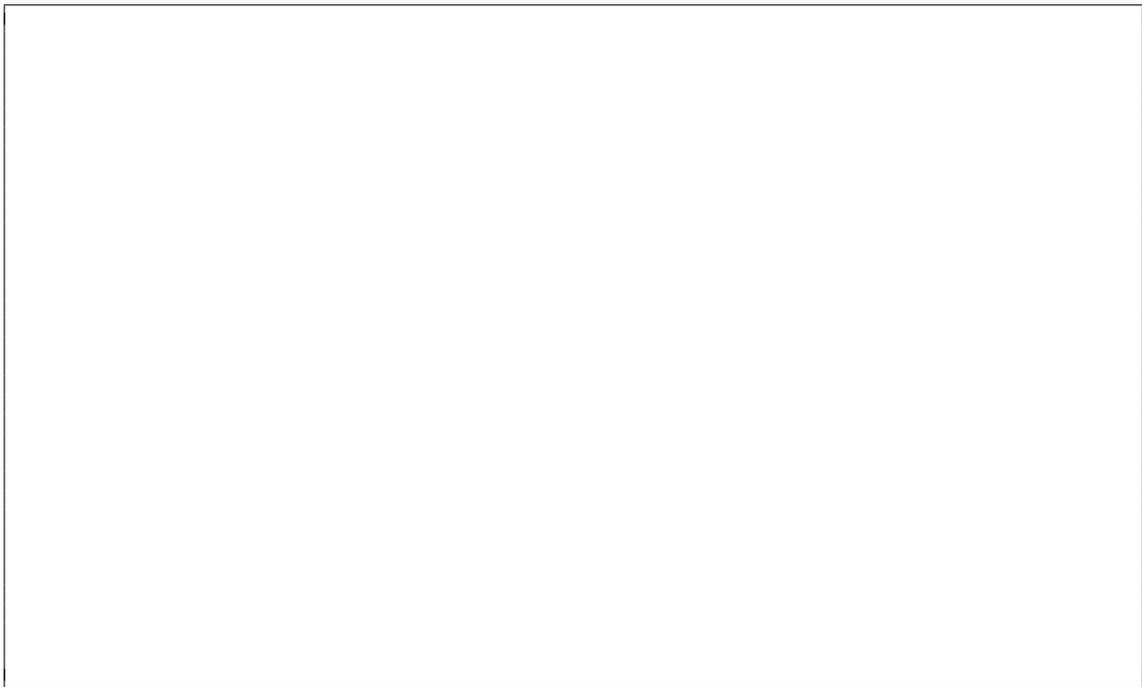
2. (20 points) Prove that for every integers $a_1, \ldots, a_n$ there are $k > 0$ and $\ell \geq 0$ such that $k + \ell \leq n$ and $\sum_{i=0}^{\ell} a_{k+i}$ is divisible by n.

3. (10 points) How many 6-digit numbers are there that have the same reminder modulo 2 of all the digits?

4. (20 points) How many pairs of subsets $A, B \subseteq [n]$ are there such that $A \cap B \neq \emptyset$.