$c(n,k) = \#$ permutations with k cycles.

$s(n,k) = (-1)^{n-k} c(n,k)$ Stirling numbers of the first kind

Theorem

$$(x)_n = \sum_{k=0}^{n} s(n,k)\, x^k$$

Theorem

$$x^n = \sum_{k=0}^{n} S(n,k)\, (x)_k$$

$(x)_k = x(x-1)\ldots(x-k+1)$

Theorem

$$c(n,k) = c(n-1,k-1) + (n-1)\, c(n-1,k)$$

$(1 2 3)(4 5) = (2 3 1)(4 5) = (4 5)(1 2 3) \ldots$

Definition

We say that a canonical cycle form of perm. is the form where all the cycles start with a largest number And cycles are ordered in the increasing order of the first elements

$(3 1 2)(5 4)$ is the can. form of

23154

Exercise

write the perm. in the can. cyc. form

$(2\ 4\ 3)\ (1\ 5)\ (6)\ (7)$

$(4\ 3\ 2)\ (5\ 1)\ (6)\ (7)$ ← Answer

Note that if I erase the par.

i.g. consider $(4\ 3\ 2\ 5\ 1\ 6\ 7)$

The can. form of $(2\ 5\ 3)\ (1\ 4)\ (6)\ (7)$ is

$(5\ 3\ 2)\ (4\ 1)\ (6)\ (7)$

$(4\ 1)\ (5\ 3\ 2)\ (6)\ (7)$ ← Answer

Consider $G: S_n \to S_n$ such that

$\pi \to$ write it in can. cycle form $\to$ erase par. $\to$

$\to$ interpret as a perm in the one-line not $\to$

$\to G(\pi)$

Example

$124635 = (1)(2)(6534) \xrightarrow{G} 126534$

Theorem

G is a bijection

Problem

Given k numbers $a_1 \dots a_k \in [n]$.

How many permutations have $a_1 \dots a_k$ in the same cycle?

Answer: $\frac{n!}{k}$

WLOG $a_1 = n$

So we are intrested in the cases when $a_2 \dots a_k$ are to the right of n