Theorem Let a_n be the number of ways to create a certain structure on a set of n-elements and b_n be the number of ways to build another structure. If F and G are the gen. functions for a_n & b_n resp. then $F \cdot G$ is the gen. function for the number of ways to split n objects into two parts and build the first structure on the first part and the second structure on the second.

Exercise

We have a company „Bolshoy Brat"
It needs to finish two project.
To do this CEO writes the list of all the employees and split it into two parts: the first works on the first project and the second works on the second.

Each group needs to ~~select~~ split into marketing team and product team. How many ways to do this?

Let $A(x)$ be the gen. function for the number of ways to do this, so $A(x)=\sum_{n\geq 0} 2^n x^n =$

$$= \frac{1}{1-2x}$$

By the theorem the gen. function for the final answer is $A^2(x) = \frac{1}{(1-2x)^2} =$

$$= \frac{1}{2}\left(\frac{d}{dx} \frac{1}{1-2x}\right) = \frac{1}{2} \frac{d}{dx} \sum_{n \geq 0} 2^n x^n =$$

$$= \frac{1}{2} \sum_{n \geq 1} 2^n x^{n-1} \cdot n = \sum_{n \geq 0} \frac{2^{n+1}}{2} x^n (n+1) =$$

$$= \sum 2^n \cdot (n+1) \cdot x^n$$

Therefore the answer is $(n+1)2^n$

$$\sum_{k=0}^{n} 2^k \cdot 2^{n-k} = \sum_{k=0}^{n} 2^n = (n+1)2^n$$

You have a line of n soliders you want to split them into squads (people in a squad are consecutive in the line) and in each squad you need to select a leader. How many ways to do this?

Assume that in the ith squad there are n_i people and I've created k squads. What is the answer in this case?

$$n_1 \cdot n_2 \cdot \ldots \cdot n_k$$

What if $n_1 \ldots n_k$ are not fixed?

$$\sum_{n_1 + \ldots n_k = n} n_1 \cdot n_2 \cdot \ldots \cdot n_k$$