

Name: _____

Pid: _____

1. (10 points) Check all the correct statements.

- The inverse of the permutation $(1, 2, 3)(4, 5)$ is $(2, 1, 3)(4, 5)$.
- There are 60 permutations of the cyclic type $(2, 0, 1)$.
- Product of the permutations 13245 and 32154 is 23154.
- The number of different strings you can get by reordering letters in the word abccc is 30.
- If you have 26 balls in 5 boxes, then there is a box with at least 6 balls.

2. (10 points) Show that if $p(n)$ denotes the number of partitions of the integer n , then

$$\sum_{n \geq 0} p(n)x^n = \prod_{k=1}^{\infty} \frac{1}{1-x^k}$$

3. (10 points) Let $f(n)$ be the number of subsets of $[n]$ in which the distance of any two elements is at least three. Find the generating function of $f(n)$.

4. (10 points) Show that any permutation is a product of cycles of length 2 (such cycles are called transpositions).